Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

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Twelfth Semiannual Status Report

December 1967

on

BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL

in the

Department of Aeronautics and Astronautics
Stanford University

under

Research Grant NsG-133-61

from the

National Aeronautics and Space Administration

This report summarizes progress during the past twelve months under a continuing research grant for the period beginning December 1966. The initial grant is based on Ref. 1, and its continuation on Ref. 2. The research is supervised by Professor I. Flügge-Lotz and Professor R. H. Cannon, Jr., Principal Investigators.

A separate financial accounting will be forwarded by the University.

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SUMMARY

The research on nonlinear and optimal attitude control of planet-pointing space vehicles continued with Mr. Boykin and with new Ph.D. candidates under Professor Flügge-Lotz (Sec. A). Basic studies of optimal control, under Professor Franklin, and satellite trajectory studies, under Professor Breakwell, also continue (Secs. A and B).

In September, Professor Cannon began a second one-year leave of absence from the University to continue as Scientific Advisor to the Chief of Staff of the Air Force. Consequently, his research remains suspended, and there is nothing to report for him on this Grant for the current period.

Because of the oral report given by Professors Flügge-Lotz and Franklin at ERC in June, this report contains information for a one-year period.

A. NONLINEAR STUDIES, OPTIMAL CONTROL

(Studies Supervised by Professor Flügge-Lotz)

1. Optimal and Suboptimal Control of a Satellite in Elliptic Orbit. Steady State Consideration. (Ph.D. Research of W. Boykin)

Four configurations of satellites and their orbits are considered here. These four configurations were chosen since they exhibit a wide variety of natural attitude motions and since they are similar to some future as well as some recent earth satellites.

Satellites (1) and (4) are similar to some of today's "stable" scientific satellites, e.g., satellites of the Orbiting Geophysical Observatory and the Tiros series. Satellite (4) is such that aerodynamic forces significantly affect its attitude motion. Satellite (2) is similar to "unstable" manned and unmanned spacecraft which will be inserted into (nearly) circular earth parking orbits for transfer to orbits about other celestial bodies. Examples of such spacecraft are Apollo and Unmanned Mars Excursion Vehicle. Satellite (3) is similar to future "unstable" military applications satellites.

The orbits of the satellites are elliptic, although for satellites (2) and (3) the eccentricity is assumed to be only 0.01.

The earth-pointing orientation, i.e., the orientation such that one body-fixed axis is parallel to the local vertical and another is normal to the orbit plane, is required for the lifetimes of the satellites. Gas jets provide the control torque.

Linear differential equations with time-varying coefficients, which include terms for the gravity torque due to an oblate earth and terms for the aerodynamic torque, are used to describe the attitude motion when a satellite is practically earth-pointing. Nonlinear equations with time-varying coefficients are used to describe the attitude motion when acquisition of the earth-pointing motion from large deviation angles ($\approx 80^{\circ}$) is considered.

Pontryagin's Maximum Principle, the necessary conditions for exact solutions of optimal bounded-phase-coordinate problems and guidelines obtained from the minimum-fuel station-keeping controls devised for single-axis systems are used in the development of the station-keeping part of the control system. The acquisition part of the control system, developed here, results in acquisition of the earth-pointing motion from large angles in the time of one-quarter orbit with comparatively little fuel expenditures.

The motions of the satellites with the developed station-keeping control systems are simulated on an analog computer; and, the performances of the system are evaluated. The nonlinear differential equations which include the developed acquisition control systems with time delays are integrated by using a digital computer. The fuel expenditures and the times of acquisition obtained from these digital computer runs are compared with those for the satellite motions described approximately by linear differential equations with the optimal controls obtained from the maximum principle.

The simulation studies together with the performance evaluations showed that the control systems are quite efficient as well as reliable. (The acquisition system is a back-up system to the station-keeping system.) The overall control system, which is very simple to realize, is given in diagram form.

A report Sudaar 322 was finished by the end of July. Copies of the report were mailed October 18, 1967.

2. The Complete Attitude Control Problem for an Earth Satellite in Elliptic Orbit. (Ph.D. Research of G. Wolske)

The steepest descent procedure of Hales (Sudaar 257) was repeated successfully on a second-order system (linearized pitch) for minimum fuel. Results were verified by comparison to true optimal trajectories generated by reverse time integration.

Work is currently being directed toward applying the results of T. E. Bullock on second variations and making necessary modifications such that they can be used to solve fuel minimum problems. Fuel minimum problems are not as easily handled by second variations as some other problems because of the nonexistence of certain partial derivatives. (Approximations offer promise.) It is also desirable to be able to incorporate the knowledge of the bang-bang nature of the control of the satellite so that the second variation method (through modification) can be used to give a better strategy for changing switching times than conventional gradient methods.

3. An addendum to Report Sudaar 271, "Minimum Time Control of a Nonlinear System," by Jose Luis Garcia Almuzara and I. Flügge-Lotz was mailed on October 18, 1967. It concerned the mathematical details for the particular case

$$\ddot{x} + \sin x = u \quad \text{and} \quad |u| \leq 1$$

An abbreviated version of this paper will appear in the Journal of Differential Equations, January 1968.

4. The Validity of Linearization in Attitude Control. (Ph.D. Research of F. Curtis)

The optimal control of two systems

$$\begin{array}{lll} \text{I.} & \ddot{x} + \sin x = u(t) & \\ & & \text{with } |u| \leq A > 1 \\ \text{II.} & \ddot{x} + x = u(t) & \end{array}$$

for the performance criterion $\int_0^T |u| \, dt \rightarrow \min$ shall be compared.

For those error and error rate disturbances which can be zeroed within 3/8 of a circular satellite orbit, Mr. Curtis has developed the complete time-varying feedback control law which minimizes the amount of fuel expended in reorienting the satellite to its earth-pointing position. By having the solution exhibited in this feedback form one is able to minimize the effect of any unforeseen disturbances or malfunctions which could occur during the reorientation.

Now under development is the determination of that time-dependent indifference curve separating the region for which we directly zero the pitch error and error rate from the region where less fuel is expended by allowing the satellite to make a complete somersault (return the pitch angle to 2π instead of zero) while zeroing the error rate.

5. Systems with a Delayed Control Input.

The work started with studying phase trajectories of the system

$$e''(\tau) + 2\zeta\omega e'(\tau) + \omega^2 e(\tau) = u(t-\tau)$$

where τ is large compared to the period of the uncontrolled system.

This investigation will be done by Mr. G. Swanson, who has still to pass his Ph.D. qualifying examination. Therefore, greater intensity cannot be put in this work before the end of January.

Research on optimal control of systems with time delay in the feedback path have been supported by the Air Force; a report based on the Ph.D. research of Dr. Ross is just being finished.* It may be interesting to incorporate results from that investigation in our new program because, in general, both delays will occur.

(Studies Supervised by Professor Franklin)

6. Computing Optimal Controls (Ph.D. Research of G. Gerson)

An important class of optimal control problems are those involving state-variable inequality constraints. Such problems arise, for example, in determining the optimal trajectory for an orbiting body to re-enter the earth's atmosphere subject to a heating constraint, and in determining optimal maneuvers for high-performance aircraft subject to structural limitations. Numerical solutions for such problems have been obtained by using necessary conditions which an optimal control must satisfy [Ref. 3]. Approximate solutions have been obtained by means of penalty functions [Ref. 4] and dynamic programming [Ref. 5]. None of these methods is ideal. The use of the necessary conditions requires that the general structure of the optimal trajectory first be assumed, and then a search must be made for a finite number of parameters. The penalty function method is

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* The contract with the Air Force expires January 1968 and will not be continued, since this problem is on our long range NASA program.

effective, but it has been reported [Ref. 6] to converge at a slower rate than the method based on necessary conditions. Dynamic programming is limited to low-order systems and requires discrete quantizing of continuous variables.

The present research is concerned with a new method for obtaining approximate solutions for optimal control problems with state-variable inequality constraints. It is not expected that this method will prove to be ideal. However, it is hoped that this research will provide new theoretical insight as well as an improved computational technique for some cases of interest.

The problem treated is the standard optimal control problem described as follows. We are given a dynamical system described by the n-vector ordinary differential equation

$$\frac{dx}{dt} = \dot{x} = f(x,u,t), \qquad x(t_o) = x_o \tag{1}$$

where x is the state of the system, t is time, and the r-vector u is the control input. The initial state x_o is specified, and the final state $x(t_f) = x_f$ must satisfy the q-vector terminal constraint

$$\psi(x_f) = 0 \tag{2}$$

The final time t_f may or may not be specified. The closed set U of admissible control values in R^r is given, and the control must be piecewise continuous and satisfy

$$u(t) \in U, \qquad t_o \leq t \leq t_f \tag{3}$$

The trajectory $x(t)$ must satisfy the state-variable inequality constraint

$$S(x(t)) \geq 0, \qquad t_o \leq t \leq t_f \tag{4}$$

where S is a real-valued differentiable function of the state x. Also given is a performance functional

$$J([u]) = \int_{t_o}^{t_f} \ell(x,u,t)\, dt + M(x(t_f)) \tag{5}$$

The problem is to determine a control function $[u(t)]$ which, when applied to the system (1), satisfies (2), (3), and (4), and minimizes (5).

The constraint (4) introduces computational difficulties not present in optimal control problems without state-variable inequality constraints. The approximation method under study replaces the above problem with an approximating problem (AP). AP is the same as the original problem except (1) and (4) are combined to form a new dynamical system

$$\dot{x} = f(x,u,t) + \alpha(S(x);K)S'_x, \quad x(t_o) = x_o \tag{6}$$

where S'_x is the gradient of $S(x)$, and α is a real-valued function of S and the parameter K with the properties:

1. $\alpha(S;K) \geq 0$.
2. For fixed K, $\alpha(S;K)$ is monotone nonincreasing with S.
3. $\lim\limits_{K\to\infty} \alpha(S;K) = \begin{Bmatrix} 0, & S \geq 0 \\ \infty & S < 0 \end{Bmatrix}$.

The approximating problem is to determine, for each K, a control $u(t;K)$ which, when applied to the system (6), satisfies (2) and (3) and minimizes (5). This research is concerned with determining the conditions under which the solutions of AP converge to the solution of the original problem as K tends to infinity.

As an example, we considered the dynamical system

$$\begin{aligned} \dot{x}_1 &= x_2, \quad x_1(0) = 1 \\ \dot{x}_2 &= u, \quad x_2(0) = 0 \end{aligned} \tag{7}$$

with $t_f = 1$ fixed, and terminal constraint

$$x_1(1) = x_2(1) = 0 \tag{8}$$

No constraint was placed on admissible control values. The trajectory was required to satisfy the state-variable inequality constraint

$$S(x) = x_2 - \ell \geq 0 \tag{9}$$

where ℓ is a fixed (negative) quantity, and the performance functional to be minimized was given by

$$J([u]) = \frac{1}{2} \int_0^1 u^2(t)\, dt \tag{10}$$

In order that the constraint (9) be meaningful, it was assumed that the solution to the unconstrained problem violates (9).

This particular problem was mainly chosen to test the feasibility of the new constraint formulation on the simplest possible nontrivial example. A physical example which would have these equations would be an inertia wheel used for attitude control. A velocity constraint is present representing the maximum possible momentum storage in the wheel. The boundary conditions on the states are more or less arbitrary, and the case chosen will illustrate the nature of any optimal trajectory.

The approximating problem dynamics are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + \frac{1}{\sqrt{K}}\, e^{-K(x_2-\ell)} \end{aligned} \tag{11}$$

The Hamiltonian for AP is

$$H = \lambda_1 x_2 + \lambda_2 \left(u + \frac{1}{\sqrt{K}}\, e^{-K(x_2-\ell)} \right) + \frac{1}{2} u^2 \tag{12}$$

Minimizing H gives $u = -\lambda_2$, and the Euler-Lagrange equations are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\lambda_2 + \frac{1}{\sqrt{K}}\, e^{-K(x_2-\ell)} \\ \dot{\lambda}_1 &= 0 \\ \dot{\lambda}_2 &= -\lambda + \sqrt{K}\, e^{-K(x_2-\ell)}\, \lambda_2 \end{aligned} \tag{13}$$

with boundary conditions $x_1(0) = 1$, $x_2(0) = x_1(1) = x_2(1) = 0$.

By careful study of the Euler-Lagrange equation (13) we were able to show that a solution exists to the approximating problem for every K and that, as K tends to infinity the solution to AP tends to a solution of the original problem. The system was programmed to see how convergence went in this case. The results were disappointing since an excessive amount of computer time was required to integrate the highly nonlinear equations (11). At the present time study of alternative constraint function is being pursued along with a more general existence proof for the method.

B. SATELLITE ORBIT STUDIES
(Studies Supervised by Professor Breakwell)

1. Long-Period Behavior of Lunar Orbiters. (Ph.D. Research of J. Vagners)

The orbit of the earth relative to the moon is idealized as circular and in the lunar equatorial plane. Zonal lunar harmonics up through J_4 are included in the analysis and short-period and medium-period effects (monthly, bi-monthly, etc.) are removed. The resulting long-period analysis is a generalization of that by Kozai [Ref. 7], in which J_3 and J_4 were neglected. Conclusions telling which orbits survive indefinitely, i.e., never impact on the lunar surface, are somewhat sensitive to the numerical values of J_3 and J_4, and sets of calculations were made based on either U.S. or U.S.S.R. announced preliminary values.

2. Motion Near Earth-Moon L_4 Libration Point. (Ph.D. Research of H. B. Schechter)

The out-of-plane solar perturbation was found to be unimportant, as was nonlinear coupling between in-plane and out-of-plane motion. The faster of the two elliptical modes of the linearized free motion is parametrically excited by the solar perturbation. There is a stable amplitude for this mode which yields a maximum excursion from L_4 of approximately 90,000 km. At this amplitude this mode is synchronized with the sun both in frequency and phase, i.e., the vehicle's motion in the rotating earth-moon frame is westward around L_4, once per lunar month (not sidereal month), with the vehicle aligned with L_4 and the sun four times a month, in fact at maximum and minimum distance from L_4. The stability of this excited mode is destroyed if the slower linearized mode is also appreciably present. The nonlinear coupling between the modes is sufficient to upset the already excited mode if the slow mode has an amplitude greater than approximately 6,000 km.

3. Station-Keeping of Vehicles at Collinear Libration Points (Ph.D. Research of R. W. Farquhar)

The possibility exists of controlling the position, near earth-moon L_2 for example, of the center of mass of two vehicles tied together by a light tether, simply by suitably controlling the length of the tether. This is primarily because the nonlinearity of the gravitational field makes the equilibrium position of the center of mass dependent on the length of the tether, the two vehicles being aligned with earth and moon. It was found that the length of the tether can be continually adjusted to simultaneously control both the relative orientation of the two vehicles and the position of their center of mass. Sample calculations involved a tether of length 4,000 km with mass only a few percent of that of two equal vehicles.

C. OTHER ACTIVITIES

Professor Cannon has continued his leave of absence to act as Scientific Advisor to the Chief of Staff of the Air Force for another year. He will fully resume his responsibilities at the University in September 1968.

Professors Cannon and Franklin attended the Joint Automatic Control Conference held in Philadelphia of this year.

In June Professors Flügge-Lotz and Franklin visited ERC to meet the staff there and to given an oral report on the state of the research supported by this Grant. At that time, it was decided that a return visit to Stanford by those ERC staff sharing a mutual interest in the subjects under investigation should be made in the Fall. Thus, would exchange of ideas and research results be greatly facilitated. In November Messrs. Schuck and Neat visited Stanford for this purpose. It is hoped that these exchanges will continue to prove fruitful in the future.

Professor Flügge-Lotz co-authored two papers on attitude control of a satellite in elliptic orbit [Ref. 9,10].

Professor Breakwell attended the AIAA meeting in New York in January, including the AIAA Astrodynamics Committee meeting. He also attended the NASA ERC Technical Reviews in Guidance Theory and Trajectory Analysis in April, September and December reporting on astrodynamics research on Grant 133-61. In June he attended the Colloquium on Advanced Problems and Methods for Space Flight Optimization in Liege, Belgium and presented a paper, "Minimum Impulse Transfer Between a Circular Orbit and a Nearby Non-Coplanar Orbit" [Ref. 11]. In October he presented a paper, "Stochastic Optimization Problems in Space Guidance," at the U.S. Army Advanced Seminar on Stochastic Optimization and Control, University of Wisconsin [Ref. 12]. In December he attended a (Classified) panel on Satellite Tracking Accuracy, at Patrick Air Force Base. Professor Breakwell served as Chairman of the First Session of the NASA Seminar on Environment Induced Orbital Dynamics held at Marshall Space Flight

Center, Huntsville, Alabama, in June. In August he served as Chairman of a session on Analytical Dynamics at the AIAA Guidance, Control and Flight Dynamics Conference held at Marshall Space Flight Center.

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